Erosion modeling

Universal Soil Loss Equation (USLE) general equation:

$$E = RKLSCP$$

where E is average annual soil loss in $ton/(acre.year) = 0.2242 kg/(m^2.year) = 2.242 ton/(ha.year)$,

R is rainfall factor in (hundreds of ft-tonf.in)/(acre.hr.year) = 17.02(MJ.mm)/(ha.hr.year),

K is soil erodibility factor in (ton acre.hr)/(hundreds of acre ft-tonf.in) =

= 0.1317(ton.ha.hr)/(ha.MJ.mm),

LS is a dimensionless topographic (length-slope) factor,

C is a dimensionless land cover factor, and

P is a dimensionless prevention measures factor.

The modified 3D factor, representing topographic potential for erosion **at a point on the hillslope**, is a function of the upslope area per unit width U and the slope angle:

$$LS = (m+1)\,(U/22.1)^m\,(\sin\beta/0.09)^n$$

where

U is the upslope area per unit width (measure of water flow) in meters (m^2/m),

β is the slope angle in degree,

22.1 is the length of the standard USLE plot in meters,

$0.09 = 9\% = 5.15°$ is the slope of the standard USLE plot.

The values of exponents range for $m = 0.2 - 0.6$ and $n = 1.0 - 1.3$, where the lower values are used for prevailing sheet flow and higher values for prevailing rill flow.

Unit Stream Power Based Erosion/Deposition model (USPED)

USPED estimates a sediment transport limited case of erosion/deposition using the concept proposed by Moore and Burch (1986). It combines the USLE/RUSLE parameters and upslope contributing area per unit width U to estimate the sediment flow T at sediment transport capacity:

$$T \approx RKCPU^m(sin\beta)^n.$$

R, K, C, P, U, β are the same as in USLE, and U, β are not normalized, because

T is an estimate of sediment flow $[kg/(m.s)]$ or $[ton.m/(ha.year)] = [ton/(10000m.year)]$.

(rather than soil loss E $[kg/(m^2.s)]$).

The net erosion/deposition D is then computed as a divergence of sediment flow (change in a 2d field representing sediment flow in the direction of elevation surface gradient):

$$D = \nabla \cdot (T\mathbf{s_0}) = \frac{\partial (T\cos\alpha)}{\partial x} + \frac{\partial (T\sin\alpha)}{\partial y},$$

where

α in degrees is the aspect of the terrain surface (direction of flow).
We get D in $[kg/(m^2s)]$ by dividing $T[kg/(ms)]/dx[m]$ or $T[ton/10000m.year]/dx[m] = D[ton/(10000m^2.year)] = D[(ton/ha.year)]$

The exponents m, n control the relative influence of water and slope terms and reflect the impact of different types of flow. The typical range of values is $m = 1.0 - 1.6, n = 1.0 - 1.3$, with the higher values reflecting the pattern for prevailing rill erosion with more turbulent flow when erosion sharply increases with the amount of water. Lower exponent values close to $m = n = 1$ better reflect the pattern of compounded, long term impact of both rill and sheet erosion and averaging over a long term sequence of large and small events.

We can transform the rate of erosion or deposition to elevation change at grid point:

$$dz = D/\varrho$$

where ϱ is soil density in $[kg/m^3]$=$[0.001ton/m^3]$ so we divide erosion/deposition rate D in $[ton/(ha.year)]$ by 10-times density in $[kg/m^3]$
$dz[m/year] = D/(10.\varrho)$ is elevation change per year at grid point.
Mass of sediment eroded or deposited at a cell

$$D_{cell}[ton/year] = D[ton/(10000m^2s)] * r^2[m^2]$$

where r is raster resolution in meters.